Intermediate Inputs and Cross-Country
Productivity Differences*

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Abstract

Agricultural productivity is important in accounting for international income differences. Both real and nominal intermediate input intensity in agriculture are highly positively correlated with agricultural productivity across countries. However, models with only exogenous price variations are not able to generate any cross-country variation of nominal intermediate input intensity. Therefore, structures that can generate variations in nominal intermediate input intensity are needed in these models. We add a minimum structure to such a model and show that intermediate inputs could account for substantially more cross-country differences in both agricultural and aggregate productivity.

Keywords: Nominal intermediate input intensity; Real intermediate input intensity; Agricultural productivity differences; Price distortions;

JEL Classification: O1; O4

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1 Introduction

Agriculture is crucial for understanding international income differences (Caselli, 2005; Restuccia, Yang and Zhu, 2008; Gollin, Lagakos and Waugh, 2014). Agricultural productivity is extremely low in poor countries in which most employment works in agriculture. Restuccia, Yang and Zhu (2008) document that the real intermediate input-output ratio in agriculture is highly positively correlated with agricultural productivity across countries. Quantitatively, they find intermediate inputs only play a small role in accounting for both agricultural and aggregate productivity differences across countries. In their model, the income share of intermediate inputs in the aggregate production technology is identical across countries and this structure generates no variation in international nominal intermediate input-output ratios in agriculture, even though there are large cross-country price distortions. However, as shown in Figure 1, the nominal intermediate input-output ratio is also highly positively correlated with agricultural productivity across countries.

Therefore, more structure is needed in Restuccia, Yang and Zhu (2008)'s model in order to be consistent with our empirical finding in Figure 1. What happens if the nominal intermediate input-output ratios in agriculture generated in the model can match their empirical counterparts? In this case, since poor countries have smaller income shares of intermediate inputs in agriculture, less intermediate inputs and more labor will be used for the agricultural good production. This implies that cross-country differences of the real agricultural intermediate input-output ratio in the model would be larger. Hence, cross-country agricultural productivity differences generated by the model would be larger as well. We illustrate the mechanism in detail in Section 2.

To quantify the effect of intermediate inputs in accounting for cross-country productivity differences, we consider a country-specific explicit distortion on intermediate inputs

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1 Donovan (2016) document that both the real and nominal intermediate input-output ratio in agriculture, are highly positively correlated with GDP per capita across countries.

2 The cross-country nominal intermediate input to output ratio and agricultural output per worker in 1985 are calculated from the data reported by Rao (1993), which is the same source of Restuccia, Yang and Zhu (2008).
in agriculture in Restuccia, Yang and Zhu (2008)’s model. We then calibrate this distortion to match the nominal intermediate input intensity in each country. Quantitatively, we find adding this structure could improve the performance of their model substantially. In particular, the modified model can account for 70 percent and 50 percent of cross-country differences in agricultural and aggregate productivity, respectively. Notice that Restuccia, Yang and Zhu (2008) accounts for only 20 percent and 30 percent of these differences.

The remainder of this paper is organized as follows. Section 2 describes the model and illustrates the intuition. Section 3 conducts quantitative analyses. Section 4 concludes.

## 2 Model

In this section, we briefly describe the model of Restuccia, Yang and Zhu (2008). Consider a two-sector economy in which each sector produces one good: the agricultural good $a$
and the non-agricultural good \( n \). The representative household has the following Stone-Geary preference:

\[
alog (c_a - \bar{a}) + (1 - a) \log c_n, \tag{1}
\]

where \( a \in (0, 1) \) is the weight parameter and \( \bar{a} \) is the subsistence requirement of the agricultural good. The households supply total labor \( N \) inelastically. Let \( Y_i \) and \( L_i \) be the output and the labor input in sector \( i \in \{a, n\} \). The production of the agricultural good uses land input \( Z \) and a non-agricultural intermediate input \( X \):

\[
Y_a = X^\alpha \left(Z^{1-\sigma} (\kappa A L_a)^\sigma\right)^{1-\alpha},
\]

where \( 0 < \sigma < 1, 0 < \alpha < 1, \kappa \) is the sector-specific productivity in agriculture, and \( A \) is the economy-wide productivity (TFP). In our quantitative analysis, the income share of intermediate inputs \( \alpha \) will be country-specific. The production technology of the non-agricultural good is linear:

\[
Y_n = AL_n.
\]

All markets are competitive and we let the non-agricultural good be the numeraire. Let \( p_a, w_a, \) and \( \pi \) denote the prices of the agricultural good, labor and the intermediate input, respectively. Due to the friction in the labor market, assume that \( w_a = (1 - \theta) w_n \), where \( 0 < \theta < 1 \) and \( w_n \) is the wage rate in non-agriculture.

To generate cross-country nominal intermediate input intensity, we add an explicit distortion \( \tau \) on intermediate inputs in agriculture. Hence, the essential price of intermediate inputs is \( (1 + \tau) \pi \).
2.1 Intuition

To illustrate how matching the nominal intermediate input intensity changes the quantitative results, consider a special case of the model. In particular, assume \( a = 0 \). In this case, only the subsistence agricultural goods consumption is needed, i.e. \( c_a = \bar{a} \). Notice that agricultural good market clearing condition requires \( Y_a = N\bar{a} \). The first-order conditions in agriculture imply

\[
\frac{X}{L_a} = \left( \frac{\alpha}{1 - \alpha} \right) \left( \frac{(1 - \theta) A}{\pi (1 + \tau) \sigma} \right).
\]  

(2)

Intuitively, as the TFP \( A \) decreases, less intermediate inputs and more labor will be used to produce \( Y_a \). This immediately implies a lower intermediate input intensity in agriculture \( \left( \frac{X}{L_a} \right) \) and agricultural productivity \( \left( \frac{Y_a}{L_a} \right) \) and a higher employment share in agriculture \( \left( \frac{L_a}{N} \right) \). Restuccia, Yang and Zhu (2008) show that the higher price of intermediate inputs \( \pi \) and the larger wage wedge \( \theta \) in poor countries amplify the effects of lower TFP.

Notice that, in general, output in agriculture \( Y_a \) decreases as TFP \( A \) becomes lower. This tends to drive up the intermediate input intensity in agriculture \( \left( \frac{X}{L_a} \right) \). However, we show the above effect is dominant in our quantitative analysis.\(^4\)

It is clearly from the following first-order condition:

\[
\frac{\pi X}{p_a Y_a} = \frac{\alpha}{1 + \tau},
\]

(3)

that cross-country nominal intermediate input intensities are identical if they share the same \( \tau \), even if price distortions \( \pi \) vary across country. However, \( \tau \) is country-specific. If we calibrate this distortion to match the nominal intermediate input intensity in each country, then \( \tau \) will be larger in poor countries than in rich ones. Equation (2) implies the amplification effect is even larger.

\(^3\)According to Restuccia, Yang and Zhu (2008)’s calibration, \( a \) is indeed very close to zero.

\(^4\)This is the case at least for both Stone-Geary CES and non-homothetic CES preferences.
3 Quantitative Analysis

3.1 Non-Homothetic CES Preference

Stone-Geary preferences imply income-dependent relative income elasticities, which are inconsistent with empirical evidence. If \( \kappa \) is country-specific, the income elasticities for poor countries are so large that equilibrium does not exist for in their model.\(^5\) Hence, our quantitative analysis is based on non-homothetic CES preferences which generates constant relative income elasticities.\(^6\)

In addition to the explicit distortion \( \tau \), another difference between our model and the one in Restuccia, Yang and Zhu (2008) is that, instead of the Stone-Geary preference in (1), we use the following non-homothetic CES preference:

\[
\omega_a C^{\frac{\nu_a}{\epsilon}} c_a^{\frac{\epsilon - 1}{\epsilon}} + \omega_n C^{\frac{\nu_n}{\epsilon}} c_n^{\frac{\epsilon - 1}{\epsilon}} = 1, \tag{4}
\]

where \( C \) is the consumer utility measured by real income, \( \mu_i > 0 \) controls the income elasticity of demand and \( \omega_i > 0 \) is a strictly positive weight parameter for good \( i \in \{a, n\} \).\(^7\)

3.2 Calibration

We assume that there is no explicit and price distortion in the US: \( \tau_{US} = 0 \) and \( \pi_{US} = 1 \). According to the first-order condition (3), we allow the explicit distortion on intermediate inputs \( \tau \) to be country specific and calibrate them to match the nominal intermediate input output ratios of each country.\(^8\)

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\(^5\)See Liao and Wang (2018). In Restuccia, Yang and Zhu (2008), the agriculture-specific productivity \( \kappa \) are set to be identical to their US counterpart across countries and equilibrium exists for each country.

\(^6\)See Comin, Lashkari and Mestieri 2017; Liao and Wang 2018. Notice that non-homothetic CES preferences are more flexible than Stone-Geary preferences in that we are now able to consider country-specific agricultural productivity \( \kappa \).

\(^7\)See Appendix (A.1) for the necessary conditions of utility maximization of the household with non-homothetic CES preferences.

\(^8\)The relative price data are reported in Rao (1993), which is the source of Restuccia, Yang and Zhu (2008).
Table 1: Calibration for Non-homothetic CES Preferences

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon )</td>
<td>0.6100</td>
<td>Estimated from the U.S. data</td>
</tr>
<tr>
<td>( \omega_a )</td>
<td>1.0000</td>
<td>Normalization</td>
</tr>
<tr>
<td>( \omega_n )</td>
<td>1497.7</td>
<td>Agricultural employment share of the U.S.</td>
</tr>
<tr>
<td>( \mu_a )</td>
<td>0.2537</td>
<td>Estimated from the U.S. data</td>
</tr>
<tr>
<td>( \mu_n )</td>
<td>0.9529</td>
<td>Estimated from the U.S. data</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.7000</td>
<td>Income share of labor in agriculture</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.4000</td>
<td>Nominal intermediate input share of the U.S.</td>
</tr>
</tbody>
</table>

\[ 1 + \tau_j = \frac{\alpha}{\pi_j X_j}, \]

where \( j \) is the country index.

Other country-specific parameters \( A, \kappa, \theta, \pi \) and \( Z/N \) are calibrated from the data of the corresponding countries.\(^9\) All other parameters are common across countries and they are calibrated to match the U.S. data. \( \sigma = 0.7 \) and \( \alpha = 0.4 \) are the same as in Restuccia, Yang and Zhu (2008). Parameters in the non-homothetic preference we use are the same as in Liao and Wang (2018). In particular, we first use the time series data of the U.S. to estimate \( \epsilon, \mu_a \) and \( \mu_n \) based on the equilibrium conditions.\(^10\) Without loss of generality, we normalize \( \omega_a \) to be 1.\(^11\) \( \omega_n \) is calibrated to match the agricultural employment share of the U.S in 1985. Table (1) summarizes the main calibrated values.

### 3.3 Quantitative Results

Table 2 shows the quantitative results. We report the ratio of model predictions of \( \left\{ \frac{X}{Y}, \frac{\pi X}{p a Y}, \frac{Y_a}{L_a}, \frac{GDP_a}{L_a}, \frac{Y}{N} \right\} \) between the richest and the poorest 5 percent countries and the level of \( \frac{L_a}{N} \) of both groups of countries. For the sake of comparison, we present what are observed in the data and the model predictions for these key variables. We solve five quantitative scenarios in our

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\(^9\)These values are the same as those in Restuccia, Yang and Zhu (2008).

\(^10\)See Appendix A.2 for details.

\(^11\)See Appendix A.3 for the justification of this normalization.
model with the non-homothetic preference.

In the baseline scenario, both the agriculture-specific productivity $\kappa$ and the income share of intermediate inputs in agriculture $\alpha$ are set to be identical to their US counterparts across countries, as Restuccia, Yang and Zhu (2008) do in their quantitative analysis. The purpose of this exercise is to show that quantitative results in our model (with the non-homothetic preference) and in Restuccia, Yang and Zhu (2008) are quantitatively comparable.

If the agriculture-specific productivity $\kappa$ in each country is identical to the one in the US, then the intermediate input intensity in poor countries are smaller than they are if $\kappa$ is country-specific. This implies a larger cross-country difference in the intermediate input intensity (2.7 fold). In the second scenario, we let the agriculture-specific productivity $\kappa$ be country-specific and calibrate $\kappa_j$ to match the agricultural output data in country $j$. As we argued in Section 2.1, country-specific $\kappa$ lead to a 0.63-fold difference in $\frac{X}{Y_a}$, compared to the 3.1-fold difference in the data. This reserve difference in real intermediate input intensity is the reason why this scenario can perfectly account for all differences in $\frac{GDP}{L_a}$. 12

Notice that cross-country differences in agricultural and aggregate productivity and value-added per worker are larger due to larger differences in agriculture-specific productivity $\kappa$.

To see how important are the two distortions on intermediate inputs in accounting for cross-country productivity differences, we let both the price distortion $\pi$ and the explicit distortion $\tau$ be country-specific. $\pi$ is calibrated to match its empirical counterpart in each country in the data. We calibrate $\tau$ to match the nominal intermediate input intensity in each country. Results are summarized in scenario (3) and (4). The price distortion can account for more of cross-country differences only in $\frac{X}{Y_a}$ and $\frac{Y_a}{L_a}$, compared with the second scenario. The explicit distortion can account for 5 to 15 percent more of cross-country

\footnote{Notice that $\frac{GDP}{L_a} = \frac{Y_a}{L_a} \left(1 - \frac{X}{Y_a}\right)$. Hence, a reserve difference in $\frac{X}{Y_a}$ means cross-country differences are larger in $\frac{GDP}{L_a}$ than in $\frac{Y_a}{L_a}$.}
Our model with explicit distortions (scenario 4) accounts for about 70 percent and 50 percent of differences in agricultural productivity (measured by output and value-added) and aggregate productivity, respectively. Notice that Restuccia, Yang and Zhu (2008) accounts for only 20 percent and 30 percent of agricultural and aggregate productivity differences across countries. The results in this scenario confirm our intuition in Section 2. In scenario (5), we allow both distortions to vary across countries. The model with both distortions accounts for all cross-country differences in real and nominal intermediate input intensity as well as agricultural productivity. However, adding price distortions in scenario (4) can not account for more differences in agricultural employment share and aggregate productivity.

### 4 Conclusion

Nominal intermediate input-output ratios in agriculture are highly positively correlated with agricultural productivity across countries. Exogenous price variations in models such as Restuccia, Yang and Zhu (2008) are not able to account for this nominal ratio, and hence underestimate the importance of intermediate inputs in explaining cross-country agricultural productivity differences. Therefore, more structure is needed in these mod-
els. We add a minimum structure in Restuccia, Yang and Zhu (2008) and show that the modified model could account for substantially more cross-country differences in both agricultural and aggregate productivity. A promising future research direction is investigating other structures that are able to generate cross-country variations in the nominal intermediate input intensity. An example is Donovan (2016) who consider incomplete markets in Restuccia, Yang and Zhu (2008)’s model with heterogeneous households.

References


Online Appendix

A Appendix

A.1 Non-Homothetic CES Preferences

The household chooses the bundle \( \{c_a, c_n\} \) to maximize index \( C \) subject to the non-homothetic constant elasticity of substitution (CES) aggregator (4) and the budget constraint

\[
p_a c_a + c_n = y,
\]

where \( y \) is the income of the household. The optimal decision must satisfy the following necessary condition:

\[
\frac{c_a}{c_n} = \left( \frac{\omega_a}{\omega_n} \right)^\varepsilon C^{\mu_a - \mu_n}.
\]

Plugging the necessary condition (5) into the non-homothetic CES aggregator, we get:

\[
\omega_a c_a \varepsilon^{-1} \left( \frac{\omega_n p_a}{\omega_a} \right) \left( \frac{c_a}{c_n} \right)^{\frac{\mu_a - \varepsilon}{\mu_a - \mu_n}} + \omega_n c_n \varepsilon^{-1} \left( \frac{\omega_n p_a}{\omega_a} \right) \left( \frac{c_a}{c_n} \right)^{\frac{\mu_a - \varepsilon}{\mu_a - \mu_n}} = 1.
\]

A.2 Calibration Details

We first estimate its preference parameters \( \varepsilon \) and \( \mu_a - \mu_n \) of the non-homothetic CES preference from the empirical counterpart to the first order condition (5) using the annual data of the U.S.:

\[
\log \frac{c_a}{c_n} = \zeta + (\mu_a - \mu_n) \log C - \varepsilon \log p_a + \nu_t,
\]

where \( \zeta \) is the intercept and \( \nu_t \) is a mean-zero random measurement error uncorrelated with the independent variables. \( C \) is the aggregate real consumption from the Penn World
We next estimate the equilibrium equation (8):

\[
\log \left[ \frac{1}{c_d^{\epsilon} + c_n^{\epsilon}} \left( p_a \left( \frac{c_d}{c_n} \right)^{\frac{1}{\epsilon}} \right) \right] = \zeta' + \frac{\mu_a - \epsilon}{\mu_a - \mu_n} \log \left[ p_a \left( \frac{c_a}{c_n} \right)^{\frac{1}{\epsilon}} \right] + \nu_t',
\]

where \( \zeta' \) is the intercept and \( \nu_t' \) is a mean-zero random measurement error uncorrelated with the independent variables. Given the value of \( \epsilon \) and \( \mu_a - \mu_n \) from (7), we can estimate the value of \( \mu_a \).

### A.3 Normalization in Calibration

Rewrite that equation (6) as follows:

\[
1 = \frac{e - \mu_n}{\omega_a^{e - \mu_n} \omega_n^{\mu_a - \mu_n}} \left\{ \frac{e - 1}{c_d^{\epsilon}} \left[ p_a \left( \frac{c_a}{c_n} \right)^{\frac{1}{\epsilon}} \right]^{\frac{\mu_a - \epsilon}{\mu_a - \mu_n}} \right. + \left. c_n^{\epsilon} \left[ p_a \left( \frac{c_a}{c_n} \right)^{\frac{1}{\epsilon}} \right]^{\frac{\mu_n - \epsilon}{\mu_a - \mu_n}} \right\} \\
= \omega_a \left( 1 - \frac{\mu_a - \epsilon}{\mu_a - \mu_n} \right) \omega_n^{\mu_n - \mu_n} \left\{ p_a \left( \frac{c_a}{c_n} \right)^{\frac{1}{\epsilon}} \left[ \frac{c_a}{c_n} \right]^{\frac{\mu_a - \epsilon}{\mu_a - \mu_n}} \right\}^{-1}
\]

From the equation above we can notice that:

1. Given the value of \( \epsilon \), parameters \( \mu_a \) and \( \mu_n \) affect equilibrium allocations only through the value of \( \frac{\mu_a - \epsilon}{\mu_a - \mu_n} \). That is, the equilibrium will be the same if \( \frac{\mu_a - \epsilon}{\mu_a - \mu_n} \) does not change. Therefore, we are able to normalize one of these two parameters.

2. Given the value of \( \epsilon \), \( \mu_a \) and \( \mu_n \), parameters \( \omega_a \) and \( \omega_n \) affect equilibrium allocations only through the value of \( \frac{\mu_a - \epsilon}{\mu_a - \mu_n} \). That is, equilibrium will be the same if \( \frac{\mu_a - \epsilon}{\mu_a - \mu_n} \) does not change. Therefore, we are able to normalize one of these two parameters.

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\(^{13}\)We use observed aggregate consumption data as proxy for the real consumption index in the case of the non-homothetic CES preference. See Comin, Lashkari and Mestieri (2017) for the detailed discussion on this estimation.